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Some cryptanalytic results on Lizard

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Abstract. Lizard is a lightweight stream cipher proposed by Hamann, Krause and Meier in IACR ToSC 2017. It has a Grain-like structure with two state registers of size 90 and 31 bits. The cipher uses a 120-bit secret key and a 64-bit IV. The authors claim that Lizard provides 80-bit security against key recovery attacks and a 60-bit security against distinguishing attacks. In this paper, we present an assortment of results and observations on Lizard. First, we show that by doing 2^{58} random trials it is possible to find a set of 2^{64} triplets (K, IV_0, IV_1) such that the Key-IV pairs (K, IV_0) and (K, IV_1) produce identical keystream bits. Second, we show that by performing only around 2^{28} random trials it is possible to obtain 2^{64} Key-IV pairs (K_0, IV_0) and (K_1, IV_1) that produce identical keystream bits. Thereafter, we show that one can construct a distinguisher for Lizard based on IVs that produce shifted keystream sequences. The process takes around $2^{51.5}$ random IV encryptions (with encryption required to produce 2^{18} keystream bits) and around $2^{76.6}$ bits of memory. Next, we propose a key recovery attack on a version of Lizard with the number of initialization rounds reduced to 223 (out of 256) based on IV collisions. We then outline a method to extend our attack to 226 rounds. Our results do not affect the security claims of the designers.

Keywords: Grain v1, Lizard, Stream Cipher

1 Introduction

Lightweight stream ciphers have become immensely popular in the cryptological research community, since the advent of the eStream project [est08]. The three hardware finalists included in the final portfolio of eStream i.e. Grain v1 [HJM07], Trivium [CP08] and MICKEY 2.0 [BD08], all use bitwise shift registers to generate keystream bits. After the design of Grain v1 was proposed, two other members Grain-128 [HJMM06] and Grain-128a were added to the Grain family mainly with an objective to provide a larger security margin and include the functionality of message authentication respectively. In FSE 2015, Armknecht and Mikhalev proposed the Grain-like stream cipher Sprout [AM15] with a startling trend: the size of the internal state of Sprout was equal to the size of its key. After the publication of [BS00], it is widely accepted that to be secure against generic Time-Memory-Data tradeoff attacks, the internal state of a stream cipher must be at least twice the size of the secret key. However the novelty of the Sprout design ensured that the cipher remained secure against generic Time-Memory-Data (TMD) tradeoff

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attacks. The smaller internal state makes the cipher particularly attractive for compact lightweight implementations. However, Sprout has been cryptanalyzed in more ways than one [Ban15, EK15, LNP15, ZG15] and so naturally there has been a lot of research going into design of secure lightweight stream ciphers.

At the FSE 2017 conference of IACR ToSC, two lightweight stream ciphers Lizard [HKM17] and Plantlet [MAM16] were proposed. While Plantlet was a re-design of Sprout after patching some existing weaknesses, Lizard was a new construction. It uses a Grain-like structure with two state registers of size 90 and 31 bits. The cipher uses a 120-bit secret key and a 64-bit IV. The authors claim 80-bit security against generic TMD tradeoff attacks, and 60-bit security against distinguishing attacks. Unlike members of the Grain family [ÅHJM11, HJM07, HJMM06], Sprout [AM15] and Plantlet [MAM16], the Key-IV mixing in Lizard is not efficiently invertible. This guarantees that even if an attacker manages to recover the internal state of Lizard, it does not lead to a key recovery attack. The authors also recommend that not more than 2^{18} keystream bits be generated from one Key-IV pair, and hence Lizard is not suitable for applications requiring encryption of large bulks of data.

In this paper we present an assortment of security results on Lizard. Our results specifically exploits some unique characteristics of the key-IV initialization function used in Lizard, one of them being that the initialization function is not one-to-one. Our results can be summarized as follows:

- We show that by performing around 2^{58} random experiments, it is possible to get 2^{64} triplets (K, IV_1, IV_2) such that the Key-IV pairs (K, IV_1) and (K, IV_2) produce identical keystream bits.

- We show that by performing only around 2^{28} random trials it is possible to obtain 2^{64} Key-IV pairs (K_1, IV_1) and (K_2, IV_2) that produce identical keystream bits.

- We show that one can construct a distinguisher for Lizard based on IVs that produce shifted keystream sequences. The process takes around $2^{51.5}$ random IV trials with each trial encryption required to produce 2^{18} keystream bits. and around $2^{76.6}$ bits of memory. As we will show, these observations imply that one must be careful while constructing has functions or MACs out of this construction.

- We first propose a key recovery attack on a version of Lizard with the number of initialization rounds reduced to 223 (out of 256) based on IV collisions. We then outline a method to extend our attack to 226 rounds. Both attacks require time complexity equal to 2^{69} encryptions and $2^{71.5}$ bits of memory.

Our work provides some insights into the issues that arise if the key-IV initialization function of a stream cipher is not one-to-one. It leaves the stream cipher open to situations where the two different key-IV pairs produce same keystream segments, which can be exploited further to mount key recovery attacks, which is exactly what we have done in this work, although on a reduced round version of Lizard. Although finding internal collision may not always leave the cipher vulnerable, it may be judicious to avoid it. The stream cipher MICKEY [BD08] also has a key-IV initialization that is not one-to-one, but the cipher is designed in such a way that it is infeasible to find distinct key-IV pairs that give rise to a collision in the internal state. Therefore, we can conclude that if at all it is necessary for a stream cipher to have an initialization function that is not one-to-one, it may be beneficial to design the cipher in a way that renders finding internal collisions practically infeasible, although internal collisions do not necessarily leave the cipher vulnerable as in the case of Lizard.

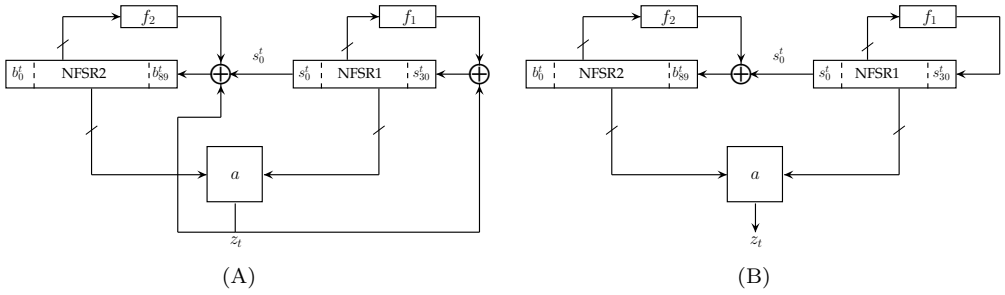


Figure 1: Block Diagram of Lizard (A) In Phase 2 of Initialization, (B) During keystream generation

1.1 Organization of the Paper

We summarize the contributions in this paper as follows. In Section 2, we present the mathematical description of the Lizard stream cipher. In Section 3, we present the algorithm to obtain key-IV pairs (K, IV_1) and (K, IV_2) produce identical keystream bits. In Section 4, we show how to obtain 2^{64} Key-IV pairs (K_1, IV_1) and (K_2, IV_2) that produce identical keystream bits. In Section 5, we show how to construct a distinguisher for Lizard based on IVs that produce shifted keystream sequences. Finally in Section 6, we propose a key recovery attack on a version of Lizard with the number of initialization rounds reduced to 223 and 226 (out of 256) based on IV collisions. In Section 7, we conclude the paper.

2 Description of Lizard

The exact structure of Lizard is explained in Figure 1. It consists of two NFSRs of size 90 and 31 bits each. Certain bits of both the shift registers are taken as inputs to a combining Boolean function, whence the keystream is produced. The 121-bit inner state of Lizard is distributed over the two NFSRs, NFSR1 and NFSR2, whose contents at time $t = 0, 1, \ldots$ is denoted by $S^t = (s_0^t, s_1^t, \ldots, s_{30}^t)$ and $B^t = (b_0^t, b_1^t, \ldots, b_{89}^t)$ respectively. We also have, for $t \in \mathbb{N} \setminus \{0, 128\}$, $s_i^{t+1} = s_{i+1}^t$ for $i = 0$ to 29 and $b_i^{t+1} = b_{i+1}^t$ for $i = 0$ to 88. The keystream is produced after performing the following steps:

Phase 1: Key-IV loading: Let $K = (k_0, k_1, \ldots, k_{119})$ denote the 120-bit key and $IV = (v_0, v_1, \ldots, v_{63})$ be the 64-bit public IV. The registers of the keystream generator are initialized as follows:

$$b_j^0 = \begin{cases} k_j \oplus v_j, & \text{for } j \in \{0, 1, 2, \ldots, 63\} \\ k_j, & \text{for } j \in \{64, 65, 66, \ldots, 89\} \end{cases}$$

$$s_i^0 = \begin{cases} k_{i+90}, & \text{for } i \in \{0, 1, 2, \ldots, 28\} \\ k_{119} \oplus 1, & \text{for } i = 29 \\ 1, & \text{for } i = 30 \end{cases}$$

Phase 2: Mixing: During this phase the cipher is clocked for 128 cycles without producing any keystream bits. During this phase the registers are updated as follows. For $t = 0, 1, 2, \ldots, 127$, we compute:

$$b_i^{t+1} = b_{i+1}^t, \quad \text{for} \quad i \in \{0, 1, \ldots, 88\}$$
$$b_{89}^{t+1} = z_t \oplus s_0^t \oplus f_2(B^t)$$

$$s_i^{t+1} = s_{i+1}^t, \quad \text{for} \quad i \in \{0, 1, \ldots, 29\}$$
$$s_{30}^{t+1} = z_t \oplus f_1(S^t)$$

where $f_1(S^t), f_2(B^t)$ and z_t are computed as follows:

$$\begin{aligned} f_1(S^t) = {} & s_0^t \oplus s_2^t \oplus s_5^t \oplus s_6^t \oplus s_{15}^t \oplus s_{17}^t \oplus s_{18}^t \oplus s_{20}^t \oplus s_{25}^t \oplus s_8^t \cdot s_{18}^t \oplus s_8^t \cdot s_{20}^t \oplus \\ & s_{12}^t \cdot s_{21}^t \oplus s_{14}^t \cdot s_{19}^t \oplus s_{17}^t \cdot s_{21}^t \oplus s_{20}^t \cdot s_{22}^t \oplus s_4^t \cdot s_{12}^t \cdot s_{22}^t \oplus s_4^t \cdot s_{19}^t \cdot s_{22}^t \oplus \\ & s_7^t \cdot s_{20}^t \cdot s_{21}^t \oplus s_8^t \cdot s_{18}^t \cdot s_{22}^t \oplus s_8^t \cdot s_{20}^t \cdot s_{22}^t \oplus s_{12}^t \cdot s_{19}^t \cdot s_{22}^t \oplus s_{20}^t \cdot s_{21}^t \cdot s_{22}^t \oplus \\ & s_4^t \cdot s_7^t \cdot s_{12}^t \cdot s_{21}^t \oplus s_4^t \cdot s_7^t \cdot s_{19}^t \cdot s_{21}^t \oplus s_4^t \cdot s_{12}^t \cdot s_{21}^t \cdot s_{22}^t \oplus s_4^t \cdot s_{19}^t \cdot s_{21}^t \cdot s_{22}^t \oplus \\ & s_7^t \cdot s_8^t \cdot s_{18}^t \cdot s_{21}^t \oplus s_7^t \cdot s_8^t \cdot s_{20}^t \cdot s_{21}^t \oplus s_7^t \cdot s_{12}^t \cdot s_{19}^t \cdot s_{21}^t \oplus s_8^t \cdot s_{18}^t \cdot s_{21}^t \cdot s_{22}^t \oplus \\ & s_8^t \cdot s_{20}^t \cdot s_{21}^t \cdot s_{22}^t \oplus s_{12}^t \cdot s_{19}^t \cdot s_{21}^t \cdot s_{22}^t \end{aligned}$$

$$\begin{aligned} f_2(B^t) = {} & b_0^t \oplus b_{24}^t \oplus b_{49}^t \oplus b_{79}^t \oplus b_{84}^t \oplus b_3^t \cdot b_{59}^t \oplus b_{10}^t \cdot b_{12}^t \oplus b_{15}^t \cdot b_{16}^t \oplus b_{25}^t \cdot b_{53}^t \oplus \\ & b_{35}^t \cdot b_{42}^t \oplus b_{55}^t \cdot b_{58}^t \oplus b_{60}^t \cdot b_{74}^t \oplus b_{20}^t \cdot b_{22}^t \cdot b_{23}^t \oplus b_{62}^t \cdot b_{68}^t \cdot b_{72}^t \oplus \\ & b_{77}^t \cdot b_{80}^t \cdot b_{81}^t \cdot b_{83}^t \end{aligned}$$

$$L_t = b_7^t \oplus b_{11}^t \oplus b_{30}^t \oplus b_{40}^t \oplus b_{45}^t \oplus b_{54}^t \oplus b_{71}^t$$

$$Q_t = b_4^t \cdot b_{21}^t \oplus b_9^t \cdot b_{52}^t \oplus b_{18}^t \cdot b_{37}^t \oplus b_{44}^t \cdot b_{76}^t$$

$$\begin{aligned} T_t = {} & b_5^t \oplus b_8^t \cdot b_{82}^t \oplus b_{34}^t \cdot b_{67}^t \cdot b_{73}^t \oplus b_2^t \cdot b_{28}^t \cdot b_{41}^t \cdot b_{65}^t \oplus b_{13}^t \cdot b_{29}^t \cdot b_{50}^t \cdot b_{64}^t \cdot b_{75}^t \oplus \\ & b_6^t \cdot b_{14}^t \cdot b_{26}^t \cdot b_{32}^t \cdot b_{47}^t \cdot b_{61}^t \oplus b_1^t \cdot b_{19}^t \cdot b_{27}^t \cdot b_{43}^t \cdot b_{57}^t \cdot b_{66}^t \cdot b_{78}^t \end{aligned}$$

$$\tilde{T}_t = s_{23}^t \oplus s_3^t \cdot s_{16}^t \oplus s_9^t \cdot s_{13}^t \cdot b_{48}^t \oplus s_1^t \cdot s_{24}^t \cdot b_{38}^t \cdot b_{63}^t$$

$$z_t = L_t \oplus Q_t \oplus T_t \oplus \tilde{T}_t$$

Phase 3: Second key Addition: After this the 120 bit key is added to the state as follows:

$$b_j^{129} = b_j^{128} \oplus k_j, \qquad \text{for } j \in \{0, 1, 2, \ldots, 89\}$$

$$s_i^{129} = \begin{cases} s_i^{128} \oplus k_{i+90}, & \text{for } i \in \{0, 1, 2, \ldots, 29\} \\ 1, & \text{for } i = 30 \end{cases}$$

Phase 4: Diffusion: During this phase the cipher is again clocked for 128 cycles without producing any keystream bit. However the feedback of the keystream bit is discontinued. Thus for $t = 129, 130, 131, \ldots, 256$, we compute:

$$b_i^{t+1} = b_{i+1}^t, \quad \text{for} \quad i \in \{0, 1, \ldots, 88\}$$
$$b_{89}^{t+1} = s_0^t \oplus f_2(B^t)$$

$$s_i^{t+1} = s_{i+1}^t, \quad \text{for} \quad i \in \{0, 1, \ldots, 29\}$$
$$s_{30}^{t+1} = f_1(S^t)$$

After Phase 4 is completed, the cipher starts producing the keystream bit z_t while following the same update rule.

3 Finding IV collisions for the same key

Phase 2 of the initialization process essentially clocks the two NFSRs for 128 cycles without producing keystream. Since the update functions of both the shift registers are of the form $x_0^t \oplus f(x_1^t, x_2^t, \ldots, x_{n-1}^t)$, the update function is one-to-one and efficiently invertible [Fre82]. As such the function F which maps the 121-bit input of Phase 2 to its output is essentially a permutation on $\mathbb{F}_2^{121}$. Since the same is true for Phase 4, the function map for this phase is also a permutation over the same domain. In fact, we present explicitly the process to invert one round of the state updates in Phases 2 and 4 (see Algorithms $P2^{-1}$ and $P4^{-1}$). The algorithms when iterated 128 times will invert the function maps of Phases 2 and 4 respectively. Before we present the algorithms, let us define $f_1(S^t) = s_0^t \oplus f_1'(s_1^t, s_2^t, \ldots, s_{30}^t)$ and $f_2(B^t) = b_0^t \oplus f_2'(b_1^t, b_2^t, \ldots, b_{89}^t)$ and the function $z(S^t, B^t) = z_t$.

Algorithm $P2^{-1}$

1. **Input**: S^t, B^t: The NFSR states at time t

2. **Output**: S^{t-1}, B^{t-1}: The NFSR states at time $t-1$

   - $s \leftarrow s_{30}^t, \quad b \leftarrow b_{89}^t$.
   - $B' = (b_0^t, b_1^t \ldots, b_{88}^t), \quad S' = (s_0^t, s_1^t \ldots, s_{29}^t)$
   - $\hat{z} = z(S', B')$;
   - $\hat{s} = s \oplus f_1'(S') \oplus \hat{z}, \quad \hat{b} = b \oplus f_2'(B') \oplus \hat{s} \oplus \hat{z}$
   - $S^{t-1} \leftarrow (\hat{s}, s_0^t, s_1^t \ldots, s_{29}^t)$
   - $B^{t-1} \leftarrow (\hat{b}, b_0^t, b_1^t \ldots, b_{88}^t)$
   - Return S^{t-1}, B^{t-1}

In Phase 3 of the initialization process, the designers set the last bit of NFSR2 i.e. s_{30}^{129} to 1. This makes the initialization process a non-injective function, so that there may be two different initial states that lead to the same 121-bit state after Phase 3. That is to say, it is possible to get a triplet K, IV_1, IV_2 so that after completion of Phase 2, the system initialized with K, IV_1 and the system initialized with K, IV_2 differ only in the last bit. Since Phase 3 adds the key to the first 120 bits and forces the last bits of both systems to 1, the internal states of both systems thereafter will be identical and they would obviously produce the same keystream bits. We call this event an IV collision. In

Algorithm $P4^{-1}$

1. **Input**: S^t, B^t: The NFSR states at time t
2. **Output**: S^{t-1}, B^{t-1}: The NFSR states at time $t-1$
   - $s \leftarrow s_{30}^t, \quad b \leftarrow b_{89}^t$.
   - $B' = (b_0^t, b_1^t \ldots, b_{88}^t), \quad S' = (s_0^t, s_1^t \ldots, s_{29}^t)$
   - $\hat{s} = s \oplus f_1'(S'), \quad \hat{b} = b \oplus f_2'(B') \oplus \hat{s}$
   - $S^{t-1} \leftarrow (\hat{s}, s_0^t, s_1^t \ldots, s_{29}^t)$
   - $B^{t-1} \leftarrow (\hat{b}, b_0^t, b_1^t \ldots, b_{88}^t)$
   - Return S^{t-1}, B^{t-1}

the original Lizard paper [HKM17], the authors had proven that, if the key K is unknown, then it would take around $2^{60.5}$ random IV trials with K to find an IV collision for K. What we show in this section is not opposed to the findings of [HKM17]. We show that by performing around 2^{58} random experiments it is possible to tabulate a set of 2^{64} IV collisions for 2^{64} specific different keys. We do not assume the unknown key setting as in [HKM17].

Our algorithm can be described as follows. Let $F : \mathbb{F}_2^{121} \rightarrow \mathbb{F}_2^{121}$ be the function which maps the 121-bit input of Phase 2 to its output. Since F is a permutation, so is F^{-1}. Let R be any 120-bit string. Consider the two 121-bit strings $R_0 = R||0$ and $R_1 = R||1$. Applying F^{-1} on each of these gives us $T_0 = F^{-1}(R_0)$ and $T_1 = F^{-1}(R_1)$. Now if there exists a triplet K, IV_0, IV_1 such that $T_0 = K[0 \text{ to } 63] \oplus IV_0 \ || \ K[64 \text{ to } 119] \ || \ 1$ and $T_1 = K[0 \text{ to } 63] \oplus IV_1 \ || \ K[64 \text{ to } 119] \ || \ 1$, then the systems initialized with K, IV_0 and K, IV_1 after Phase 3 will both lead to the internal state $R \oplus K||1$. The states for both systems are identical hereafter and so they produce identical keystream bits. We put the above ideas into the form of an algorithm as follows:

Algorithm to generate IV Collision

1. Set Success $\leftarrow 0$
2. Do the following till Success =1
   - Select $R \xleftarrow{R} \{0,1\}^{120}$ randomly.
   - Define $R_0 := R||0$ and $R_1 := R||1$
   - Compute $T_0 = F^{-1}(R_0)$ and $T_1 = F^{-1}(R_1)$
   - If $T_0[64 \text{ to } 119] = T_1[64 \text{ to } 119]$ and $T_0[120] = T_1[120] = 1$ then set Success =1
   - If Success =1 then exit from loop else continue.
3. Select $\alpha \xleftarrow{R} \{0,1\}^{64}$ randomly.
4. Set $K = \alpha \ || \ T_0[64 \text{ to } 118] \ || \ T[119] \oplus 1$, Set $IV_0 = \alpha \oplus T_0[0 \text{ to } 63]$ and $IV_1 = \alpha \oplus T_1[0 \text{ to } 63]$
5. Return K, IV_0, IV_1.

The above subroutine can be described as follows: we select a 120-bit string R and run the F^{-1} function on $R||0$ and $R||1$ (F^{-1} may be computed efficiently by 128 runs of $P2^{-1}$) to get the 121-bit strings T_0 and T_1 respectively. We stop only if:

A. The 64th to 119th bits of T_0 and T_1 are identical. These bits of initial state are composed with the last 56 bits of the secret key. So if T_0 and T_1 are to come from the initialization with the same key, the 64th to 119th bits need to be identical.

B. The last bit of both T_0 and T_1 is equal to 1. This is because in Phase 1, the starting state is initialized with the last bit equal to 1.

Both these events would be satisfied with probability 2^{-58} for a random R, and so the loop needs to be iterated around 2^{58} times before Success. Once the algorithm has the required pair T_0, T_1, we can make not one but 2^{64} triplets K, IV_1, IV_2 such that K, IV_1 and K, IV_2 will lead to an IV Collision. This is because the first 64 bits of the initial state is the bitwise xor of the IV and first 64 keybits. So we take any random 64-bit string α and set $K = \alpha \,||\, T_0[64 \text{ to } 118] \,||\, T[119] \oplus 1$ (the last bit is inverted because the specifications of Phase 1 indicate that the last key bit is inverted during the initialization). Then by setting $IV_0 = \alpha \oplus T_0[0 \text{ to } 63]$ and $IV_1 = \alpha \oplus T_1[0 \text{ to } 63]$ we ensure that after Phase 1, we have the required values of the initial states equal to T_0 and T_1. Since any value of α can be used, this gives us a set of 2^{64} triplets.

4 K_0, IV_0 and K_1, IV_1 that produce same keystream

Since Lizard uses an internal state of 121 bits and the key and IV in total is 184 bits long, it seems inevitable that there would exist two Key-IV pairs K_0, IV_0 and K_1, IV_1 that would lead to identical internal states after Phase 3, and hence produce exactly the same keystream bits. We call this event a Key-IV Collision. In this section, we will show that it is possible to find a Key-IV Collision after performing around 2^{28} random experiments. We will use the following subroutine:

Algorithm to generate Key-IV Collision

1. Set Success $\leftarrow 0$ and Fix a value of $L \xleftarrow{R} \{0,1\}^{56}$.
2. Do the following till Success =1
   - Select $M \xleftarrow{R} \{0,1\}^{64}$ randomly and define $R := M \,||\, L \,||\, 1$
   - Compute $S = F(R)$
   - Let $\hat{S} = S[64 \text{ to } 119]$, store $\hat{S}$ in a hash table along with current value of M.
   - If there is a collision in the hash table then Success =1.
   - If Success =1 then exit from loop else continue.
3. Let M_0 and M_1 be the values of M which result in collision.
4. That is, 64th to 119th bits of $S_0 = F(M_0||L||1)$ and $S_1 = F(M_1||L||1)$ are equal.
5. Select $\alpha \xleftarrow{R} \{0,1\}^{64}$ randomly and define $\Delta := S_0[0 \text{ to } 63] \oplus S_1[0 \text{ to } 63]$
6. Set $K_0 = \alpha \,||\, L[0 \text{ to } 54] \,||\, L[55] \oplus 1$, Set $IV_0 = \alpha \oplus M_0$.
7. Set $K_1 = \alpha \oplus \Delta \,||\, L[0 \text{ to } 54] \,||\, L[55] \oplus 1$, Set $IV_1 = \alpha \oplus \Delta \oplus M_1$
8. Return K_0, IV_0 and K_1, IV_1.

Table 1: Key-IV pairs that produce identical keystream bits

$Key - IV$	Keystream
K_0: 0000 0000 0000 0000 6850 8c64 c649 74 IV_0: 724b b286 2f5c f1b2	23f4 9770 0a91 3089 d800 5513 58e1 6352 ...
K_1: 1e45 1adc 2ad8 3124 6850 8c64 c649 74 IV_1: 3e18 82d1 d5ac 0376	23f4 9770 0a91 3089 d800 5513 58e1 6352 ...

The above algorithm can be described thus. We fix a 56-bit constant L which we will use to construct the 64th to 119th bits of the initial state. Then we choose a 64-bit constant M randomly and use it to construct the 1st 64 bits of the internal state. We run the state update function F of Phase 2 on $M \,||L\,||\,1$ and store the result in the variable S. We take the 56-bit value $\hat{S}$ which is the 64th to 119th bit of S and store it in a hash table along with the value of M. We keep doing this until we find a collision. Since we are looking for a collision over a 56-bit space, by birthday arguments this part of the algorithm should yield Success in around $\sqrt{2^{56}} = 2^{28}$ trials.

Once we have a collision we proceed as follows. Let M_0 and M_1 be the values of M that produce a collision. Then we will have the 64th to 119th bits of $S_0 = F(M_0||L||1)$ and $S_1 = F(M_1||L||1)$ equal. Phase 3 will set the 120th bit of both systems to 1, and so it is the first 120 bits we need to concentrate on. For identical keystream bits, we need that the states of both systems after the key addition of Phase 3 be equal. The 64th to 119th bit of S_0 and S_1 are already equal, so we need that the difference of the two keys K_0 and K_1 (in bits 0 to 63) that are to be added to S_0 and S_1 be equal to $\Delta = S_0[0 \text{ to } 63] \oplus S_1[0 \text{ to } 63]$. This ensures that both systems have identical internal states after Phase 3. So again, we take any 64-bit constant α and set $K_0 = \alpha \,||\, L[0 \text{ to } 54] \,||\, L[55] \oplus 1$ and $K_1 = \alpha \oplus \Delta \,||\, L[0 \text{ to } 54] \,||\, L[55] \oplus 1$. We must now ensure that the IVs be chosen so that the 2 systems start with the initial states $M_0 \,||\, L \,||\, 1$ and $M_1 \,||\, L \,||\, 1$ respectively. This can be done by setting $IV_0 = \alpha \oplus M_0$ and $IV_1 = \alpha \oplus \Delta \oplus M_1$. In Table 1 we tabulate a class of Key-IVs that produce the same keystream bits, that were found using the procedure listed above. Note that we can take any 64-bit constant α and add it to the first 64 bits of both the Keys and the IVs to get another set of Key-IV pairs that produce the same keystream bits. Thus we have 2^{64} such pairs from one run of the above algorithm.

4.1 Discussion

To explain the significance of this result and the one in the previous section, we can try to compare our findings to the case of an ideal stream cipher. If an attacker executes the following procedures on an ideal stream cipher:

1. He chooses a key and IV randomly.
2. He generates keystream bits using the key-IV pair.
3. He then stores keystream bits in a table.
4. He repeats the above process either with same key and different IVs to find IV Collisions or different key-IV pairs to find Key-IV Collisions .

For an ideal stream cipher with size of internal state equal to 121 bits, given 2^x trials, the number of collisions is expected to be around 2^{2x-121}. If $2^x = 2^{58}$, the number of collisions is 2^{-5} and in the second case when $2^x = 2^{28}$ the number of collisions is 2^{-65} on average. On the other hand, for Lizard, for both cases we can find 2^{64} collisions. It is a significantly large number.

Secondly, stream cipher update functions have been often used as compression functions in hash function contructions [CGN06, AHMNP13]. So one must be very careful while using Lizard as a component in Hash function and MACs. Since we show 2^{64} collisions in 2^{28} time, this means that the Lizard permutation may not be suitable for use in certain schemes like the Merkel-Damgård (MD) construction. To understand why, let P be the initialization function (comprising of Phases 1 to 4) used in Lizard. Imagine a scenario, in which a one-way variant of P is used as the compression function of the MD scheme. Which is to say we initialize the construction with the IV and the first message block is used in the role of the key. In order to make the function one-way, we could feed forward the state after phase 3 to the output of P. For every successive message block, we operate P by simply xoring the message block to the initial state as in Phase 1 and 3 (ignoring the IV), and running Phase 2, 4 as usual. Now, if (K_0, IV_0) and (K_1, IV_1) is a collision for P that we found in 2^{28} trials, then the message-IV pair $(K_0||M, IV_0)$ and $(K_1||M, IV_1)$ form a collision for the construction (where M is any sequence of message blocks). Since a generic collision can be found for any hash function in $2^{h/2}$ queries (where h is the size of the hash output), this implies that the digest size in the above scheme can not exceed 56 bits.

5 Distinguisher based on Shifted keystream bits

In [HKM17], the authors had proven that, if the key K is unknown, then it would take around $2^{60.5}$ random IV trials with K to find an IV collision for K. In this section, we show that even if the key is secret, (as is the setting followed in a chosen-IV distinguisher) then it takes much lesser number of trials to find IVs which produce shifted keystream bits when used with the given secret key. Before we outline our algorithm, let us look to the following theorem concerning shifted keystream bits in Lizard.

Theorem 1. *Let p be an integer greater than zero. Then, for every 120-bit secret key K in* Lizard*,*

1. *There exists around 2^6* IV Collisions *on average,*

2. *There exists around 2^7 IV pairs (IV_0, IV_1) on average, such that the key-IV pairs K, IV_0 and K, IV_1 produce exactly p-bit shifted keystream sequences.*

Proof. The proof is by construction. Let us define $G : \mathbb{F}_2^{121} \to \mathbb{F}_2^{121}$ to be the function that maps the input of Phase 4 of Lizard to its output (note that G^{-1} can be computed efficiently by iterating the Algorithm $P4^{-1}$ a total of 128 times). Consider the following subroutine:

Input: A 121-bit string U, a 120-bit key K, Output: The values 0/1/2.
Subroutine $\theta(U, K)$

1. Compute $\hat{U} = (K||0) \oplus G^{-1}(U)$.
2. If $\hat{U}[120] = 0$ then abort and return 0.
3. Compute $U'_0 = F^{-1}(\hat{U}[0 \text{ to } 119] \,||\, 0)$
4. Compute $U'_1 = F^{-1}(\hat{U}[0 \text{ to } 119] \,||\, 1)$
5. Set $r \leftarrow 0$.
6. If $U'_0[64 \text{ to } 120] = K[64 \text{ to } 118] \,||\, K[119] \oplus 1 \,||\, 1$, increment $r \leftarrow r + 1$.
7. If $U'_1[64 \text{ to } 120] = K[64 \text{ to } 118] \,||\, K[119] \oplus 1 \,||\, 1$, increment $r \leftarrow r + 1$.
8. Return r.

The above subroutine θ takes as input a 121-bit string U, a 120-bit key K and finds if the string U is a valid internal state at the beginning of the keystream generation phase (i.e. end of Phase 4) when used with the key K. In other words it finds out if there exists an IV such that K, IV leads to the internal state U after the four phases of initialization. The subroutine first peels off the effect of Phase 4 and 3 by applying G^{-1} and adding K to obtain $\hat{U}$. Since Phase 3 of the forward initialization process sets the last bit to 1, the last bit of a valid initialization must result in $\hat{U}[120] = 1$, failing which the subroutine returns 0. After this, the algorithm runs F^{-1} on both $\hat{U}[0 \text{ to } 119] \,||\, 0$ and $\hat{U}[0 \text{ to } 119] \,||\, 1$ to get U'_0 and U'_1 respectively (since Phase 3 sets the last bit automatically to 1, both $\hat{U}[0 \text{ to } 119] \,||\, 0$ and $\hat{U}[0 \text{ to } 119] \,||\, 1$ are candidates for valid states at this point). The last 57 bits of either U'_0 and U'_1 have to be equal to $K[64 \text{ to } 118] \,||\, K[119] \oplus 1 \,||\, 1$ for a valid initialization, and the subroutine returns 2 if both the conditions in lines 6, 7 are met, and 1 if only one condition is met. Otherwise the subroutine returns 0. Note that we do not need to impose a similar condition on the first 64 bits, since these are supposed to be the bitwise xor sum of the key and the IV. So whenever either one or both conditions in Lines 6 or 7 are satisfied, $U'_i[0 \text{ to } 63] \oplus K[0 \text{ to } 63]$ (for $i = 0$ or 1 or both), gives us the value of the IV, that along with K leads to the state U after Phase 4.

One can use the above subroutine to estimate the number of IV Collisions for a single key K. It is given as the number of times $\theta(U, K)$ returns 2, when U is iterated over all the possible 2^{121} values. Note that for the subroutine to return 2, a total of 115 bit conditions need to be satisfied, one in Line 2 and 57 each in Lines 6, 7. Assuming that these bit conditions are satisfied according to $i.i.d$ uniform distributions, the total number of times the subroutine returns 2 can be estimated as $2^{121-115} = 2^6$. Thus on average, for each K there exist 2^6 IV pairs that collide.

We can also use the algorithm to estimate the number of IV pairs that result in exactly p-bit shifted keystream sequences (for $p > 0$). Let $g : \mathbb{F}_2^{121} \rightarrow \mathbb{F}_2^{121}$ that maps the transition resulting from one clock cycle in Phase 4 (which is also the state update during the keystream generation phase). Note that therefore $G = g^{128}$. To estimate the number of such pairs we need to find the number of times $\theta(U, K)$ and $\theta(g^p(U), K)$ both return non-zero values. The probability that $\theta(U, K)$ gives a non zero value is given as (we denote

by A the event the condition in Line 2 is satisfied and the routine returns 0, B by the event when the condition in Line 6 is satisfied and C by the event when the condition in Line 7 is satisfied)

$$\begin{aligned}\mathsf{Pr}[\theta(U,K) \neq 0] &= \mathsf{Pr}[\theta(U,K) \neq 0 \mid A] \cdot \mathsf{Pr}[A] + \mathsf{Pr}[\theta(U,K) \neq 0 \mid A^c] \cdot \mathsf{Pr}[A^c] \\ &= 0 \cdot \frac{1}{2} + \mathsf{Pr}[B \vee C \mid A^c] \cdot \frac{1}{2} \\ &= \frac{1}{2} \cdot (\mathsf{Pr}[B \mid A^c] + \mathsf{Pr}[C \mid A^c] - \mathsf{Pr}[B \wedge C \mid A^c]) \\ &\approx \frac{1}{2} \cdot (2^{-57} + 2^{-57}) = 2^{-57}\end{aligned}$$

Assuming that $\theta(U,K)$ and $\theta(g^p(U),K)$ are identically and uniformly distributed, the probability that both return non-zero is $2^{-2\cdot 57} = 2^{-114}$, and so the number of IV pairs that result in p-bit shifted keystream sequences, for a given K, is $2^{121-114} = 2^7$ on average. The proof depends on the assumption that $\theta(U,K)$ and $\theta(g^p(U),K)$ are identically and uniformly distributed. This is a fair assumption to make since the key-IV mixing in the full version in Lizard is adequate, we can assume that the function mapping the state before initialization and after initialization is a PRF, and hence the reason for the assumption. □

The authors of Lizard recommend that a single Key-IV pair be used to generate not more than 2^{18} keystream bits. For any fixed K, imagine the space of initial vectors as an undirected graph $G = (W, E)$, where $W = \{0,1\}^{64}$ is the vertex set which contains all the possible 64-bit IVs as nodes. An edge $(IV_1, IV_2) \in E$ if and only if (K, IV_1) and (K, IV_2) produce either an IV collision or p-bit shifted keystream sequence (for $1 \leq p \leq 2^{18} - 121$). From the above discussion, it is clear that the cardinality of edge-set E is expected to be $(2^{18} - 121) \cdot 2^7 + 2^6 \approx 2^{25}$. So we can formulate a distinguisher as follows

1. Generate 2^{18} keystream bits $[z_0,\ z_1, \ldots, z_{2^{18}-1}]$ for the unknown key K and some randomly generated initial vector IV.

2. For $i = 0$ to $2^{18} - 121$
   - Store $[z_i, z_{i+1}, \ldots, z_{i+120}]$ in a Hash table along with the IV that generated it (a total of $121 + 64 = 185$ bits are stored).

3. Continue the above steps with more randomly generated IVs till we obtain two initial vectors for K that generate either IV Collision or p-bit shifted keystream for some p with $1 \leq p \leq 2^{18} - 121$.

The question now remains how many random IVs do we need to try before we get a match. When we run the Distinguisher algorithm for N different IVs, we effectively add $\binom{N}{2}$ edges to the coverage and a match occurs when one of these edges is actually a member of the edge-set E. Since there are potentially $\binom{2^{64}}{2}$ edges in the IV space, by the Birthday bound, a match will occur when the product of $\binom{N}{2}$ and the cardinality of E which is around 2^{25} is equal to $\binom{2^{64}}{2}$. From this equation solving for N, we get $N \approx 2^{51.5}$. This gives a bound for the time and memory complexity of the Distinguisher. The time complexity is around $2^{51.5}$ encryptions with different IVs, and the memory required is $2^{51.5} \cdot (2^{18} - 121) \cdot 185 \approx 2^{77}$ bits.

5.1 Decreasing the Memory Complexity:

We can obtain better bounds on memory if we restrict the range of p. Suppose the distinguisher uses an upper bound P. In that case, cardinality of E is around $P \cdot 2^7$. The equation we need to solve to get N becomes

$$\binom{N}{2} \cdot P \cdot 2^7 = \binom{2^{64}}{2} \Rightarrow N \approx \sqrt{\frac{2^{121}}{P}}$$

Thus the time complexity is $\sqrt{\frac{2^{121}}{P}}$ encryptions and memory required is $\sqrt{\frac{2^{121}}{P}} \cdot (P-121) \cdot 185$ bits. For example for $P = 2^{11}$, the time complexity is 2^{55} and memory complexity is $2^{73.4}$ bits.

5.2 Further Discussion

Note that when $P = 0$, i.e. when we only consider IV collisions, this reduces to the chosen-IV distinguisher already mentioned by the authors of Lizard in [HKM17]. Note also that when $P = 0$, we do not need to generate 2^{18} keystream bits for each key, IV pair since 121 keystream bits will be sufficient to observe the collision. In particular, our result is a generalization of the chosen-IV distinguisher of [HKM17] in the sense that whereas the designers consider only IV collisions, we additionally use shifted key streams for distinguishing attacks. However the complexity of the attack is not better than a classical TMD-TO distinguishers (like those of Babbage/Golic). Thus, our results reveal that even if shifted keystreams are additionally used with IV collisions, the distinguishing attack does not work well against Lizard-type construction. However, these are helpful to understand the security of the Lizard-like construction.

A TMD distinguisher on Lizard would work as follows: the attacker stores around $2^{60.5}$ pairs of 121-bit internal state and the corresponding 121-bit keystream segment that it produces in a hash table. In the online stage, the attacker needs to try to produce keystream from around $2^{121-60.5} = 2^{60.5}$ random IVs (with some unknown key) to get a match in the table, and can recover the internal state corresponding to a keystream segment. It is clear that the distinguisher we provide has computational complexity worse than a classical TMD attack. However our analysis provides some insight to the working of such structures. Even if the designer can prevent IV collision by not setting the last bit Phase 3 equal to 1 (or by expanding the state by one bit as in Plantlet), the analysis for finding shifted keystream bits still holds. The analysis provides a method to get two IVs (for an unknown key) that produces the same internal state, albeit at different clock cycles during the keystream stage. Such IV pairs can not be obtained in the classical TMD attack. Consider a MAC scheme in which, the Lizard keystream is used to generate a tag for a given message under a key, IV. The message is assumed to be xored to the internal state in some manner and the tag is produced as a function of the keystream (we have already seen constructions like [RS16] do this). If K, IV_1 and K, IV_2 produce a collision t cycles apart, then this means that we have a collision for K, IV_1, M and $K, IV_2, 0^t||M$, where M is any message string. Since the method takes around $2^{51.5+18} \approx 2^{69.5}$ hash insertions, this shows that the length of the tag should be more than 140 bits.

Of course such an attack can be prevented by length padding. But depending on the actual algebraic structure of the message addition, the attacker may find ways around this. For example, the attacker could try and find fixed points for the internal state, i.e. message strings λ for which the internal state is preserved before and after incorporating the message into the state (for example, in [Ban15], the author uses SAT solvers to find fixed points in Sprout to find keystream of short period). Then λ^i can be used as a prefix to get the lengths of $0^t||M$ and $\lambda^i||M$ to match. Thus, a designer must take into consideration these points before constructing a digest out of this structure.

6 Impossible Collision attack

In this section, we present an attack on round reduced Lizard stream cipher in which Phase 2 is reduced to 95 (out of 128) rounds, and Phase 4 is run for the full 128 rounds. The attack is similar to Impossible Differential attacks in the context of block ciphers. In impossible differential attack on a block cipher, the attacker uses an input and output differential which never occurs in the plaintext-ciphertext pairs produced by the cipher. If the impossible differential characteristic involves only a fraction of the keybits, the attacker can discard all those candidate keys that result in the characteristic, and hence reduce the size of the keyspace. An impossible collision attack follows roughly the same idea. From Theorem 1, we know that for any key K, there exist on average 2^6 pairs of IVs that produce identical keystream bits. This should also hold in the round reduced version of Lizard in which Phase 2 is reduced to 95 rounds. The attacker first exhausts the entire codebook of the 64-bit IVs to obtain 2^{64} sets of keystream sequences generated by the secret key and each of the IVs. This, therefore, takes time equivalent to 2^{64} encryptions. On average, he is expected to find 2^6 pairs of IVs that generate identical keystream bits.

Let $IV_0 = [v_0, v_1, v_2, \ldots, v_{63}]$ and $IV_1 = [\hat{v}_0, \hat{v}_1, \hat{v}_2, \ldots, \hat{v}_{63}]$ be one of the IV-pairs that result in an IV collision for the given secret key $K = [k_0, k_1, k_2, \ldots, k_{119}]$ in round reduced Lizard. Then we know that after 95 rounds of Phase 2, the key-IV pairs K, IV_0 and K, IV_1 will lead respectively to the internal states B^{95} and $\hat{B}^{95}$, which would differ only in the 120th bit. Using a computer algebra software like SAGE [Dev17], we can compute the algebraic expression for $B^{95}[0]$, i.e. the 0th bit of B^{95}. It is given as :

$$
\begin{aligned}
B^{95}[0] = \bigoplus_{i \in \mathbf{A}} x_i \oplus \quad & x_6 \cdot x_{24} \cdot x_{32} \cdot x_{48} \cdot x_{62} \cdot x_{71} \cdot x_{83} \oplus x_7 \cdot x_{33} \cdot x_{46} \cdot x_{70} \oplus x_8 \cdot x_{64} \oplus \\
& x_9 \cdot x_{26} \oplus x_{11} \cdot x_{19} \cdot x_{31} \cdot x_{37} \cdot x_{52} \cdot x_{66} \oplus x_{13} \cdot x_{87} \oplus x_{14} \cdot x_{57} \oplus x_{15} \cdot x_{17} \oplus \\
& x_{18} \cdot x_{34} \cdot x_{55} \cdot x_{69} \cdot x_{80} \oplus x_{20} \cdot x_{21} \oplus x_{23} \cdot x_{42} \oplus x_{25} \cdot x_{27} \cdot x_{28} \oplus x_{30} \cdot x_{58} \oplus \\
& x_{39} \cdot x_{72} \cdot x_{78} \oplus x_{40} \cdot x_{47} \oplus x_{43} \cdot x_{68} \cdot x_{96} \cdot x_{119} \oplus x_{49} \cdot x_{81} \oplus x_{53} \cdot x_{104} \cdot x_{108} \oplus \\
& x_{60} \cdot x_{63} \oplus x_{65} \cdot x_{79} \oplus x_{67} \cdot x_{73} \cdot x_{77} \oplus x_{82} \cdot x_{85} \cdot x_{86} \cdot x_{88} \oplus x_{98} \cdot x_{111}
\end{aligned}
$$

where $\mathbf{A} = \{5, 10, 12, 16, 29, 35, 45, 50, 54, 59, 76, 84, 89, 95, 118\}$ and the x_i's are defined as:

$$
x_i = \begin{cases} k_i \oplus v_i, & \text{for } i \in \{0, 1, 2, \ldots, 63\} \\ k_i, & \text{for } i \in \{64, 65, 66, \ldots, 118\} \\ k_i \oplus 1, & \text{for } i = 119 \end{cases}
$$

The expression consists of 38 monomials and involves 83 bits of the secret key and 50 bits of the IV. Let us now look at the algebraic expression for $B^{95}[0] \oplus \hat{B}^{95}[0]$ which is given as follows:

$$
\begin{aligned}
B^{95}[0] \oplus \hat{B}^{95}[0] = & \bigoplus_{i \in \overline{\mathbf{A}}} (v_i \oplus \hat{v}_i) \oplus (x_7 \cdot x_{33} \cdot x_{46} \oplus \hat{x}_7 \cdot \hat{x}_{33} \cdot \hat{x}_{46}) * x_{70} \oplus (v_8 \oplus \hat{v}_8) * x_{64} \oplus \\
& (x_9 \cdot x_{26} \oplus \hat{x}_9 \cdot \hat{x}_{26}) \oplus (x_{11} \cdot x_{19} \cdot x_{31} \cdot x_{37} \cdot x_{52} \oplus \hat{x}_{11} \cdot \hat{x}_{19} \cdot \hat{x}_{31} \cdot \hat{x}_{37} \cdot \hat{x}_{52}) * x_{66} \oplus \\
& (v_{13} \oplus \hat{v}_{13}) * x_{87} \oplus (x_{14} \cdot x_{57} \oplus \hat{x}_{14} \cdot \hat{x}_{57}) \oplus (x_{15} \cdot x_{17} \oplus \hat{x}_{15} \cdot \hat{x}_{17}) \oplus \\
& (x_{18} \cdot x_{34} \cdot x_{55} \oplus \hat{x}_{18} \cdot \hat{x}_{34} \cdot \hat{x}_{55}) * x_{69} \cdot x_{80} \oplus \\
& (x_{20} \cdot x_{21} \oplus \hat{x}_{20} \cdot \hat{x}_{21}) \oplus (x_{23} \cdot x_{42} \oplus \hat{x}_{23} \cdot \hat{x}_{42}) \oplus (x_{25} \cdot x_{27} \cdot x_{28} \oplus \hat{x}_{25} \cdot \hat{x}_{27} \cdot \hat{x}_{28}) \oplus \\
& (x_{30} \cdot x_{58} \oplus \hat{x}_{30} \cdot \hat{x}_{58}) \oplus (v_{39} \oplus \hat{v}_{39}) * x_{72} \cdot x_{78} \oplus (x_{40} \cdot x_{47} \oplus \hat{x}_{40} \cdot \hat{x}_{47}) \oplus \\
& (v_{43} \oplus \hat{v}_{43}) * x_{68} \cdot x_{96} \cdot x_{119} \oplus (v_{49} \oplus \hat{v}_{49}) * x_{81} \oplus (v_{53} \oplus \hat{v}_{53}) * x_{104} \cdot x_{108} \oplus \\
& (x_{60} \cdot x_{63} \oplus \hat{x}_{60} \cdot \hat{x}_{63}) \oplus (x_6 \cdot x_{24} \cdot x_{32} \cdot x_{48} \cdot x_{62} \oplus \hat{x}_6 \cdot \hat{x}_{24} \cdot \hat{x}_{32} \cdot \hat{x}_{48} \cdot \hat{x}_{62}) * x_{71} \cdot x_{83},
\end{aligned}
$$

where $\overline{\mathbf{A}} = \{5, 10, 12, 16, 29, 35, 45, 50, 54, 59\}$ and $\hat{x}_i := k_i \oplus \hat{v}_i$ for $i \in \{0, 1, 2, \ldots, 63\}$. We can see from the above equation, that $B^{95}[0] \oplus \hat{B}^{95}[0]$ is a function of 51 keybits only. This gives us an opportunity to reduce the keyspace. We start with any colliding pair of IVs. We know that for the correct key K, the first 120 bits of B_{95} and $\hat{B}_{95}$ are equal. In particular, we concentrate our attention on $\delta := B^{95}[0] \oplus \hat{B}^{95}[0]$. For the correct guess of key, δ must be zero. So if any candidate key results in $\delta = 1$, we can immediately discard it, since a collision is impossible for this candidate key. Hence the name Impossible Collision. Moreover, δ depends on only 51 key bits, so we have the added advantage of searching over only a limited keyspace. Our algorithm is as follows:

Impossible Collision Attack

1. Given around 2^6 colliding pair of IVs.
2. For each guess of the 51-bit key
   - Compute $\delta = B^{95}[0] \oplus \hat{B}^{95}[0]$ for the next colliding IV pair.
   - If $\delta = 1$, reject the key and start with another key guess **else** go to the previous step and try out another colliding IV pair.

So for each guess of the key, we compute δ for each of the 64 colliding IV pairs, and reject immediately if $\delta = 1$ for any pair. The correct key guess will give $\delta = 0$ for all colliding pairs, whereas any incorrect keybit survives all the 64 filters with a probability of 2^{-64}. And since the keyspace we are searching in has only 2^{51} candidates, it is very likely that any incorrect guess gets rejected in the process.

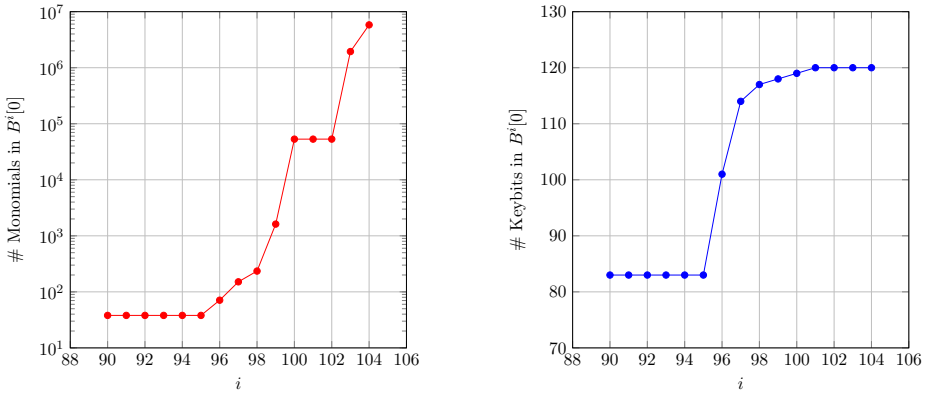
Note that it may be possible, that for certain values of IV_0, IV_1, δ is identically 0, which makes these IV pairs unusable for key filtering. This happens when the difference between IV_0, IV_1 is zero in all the 41 bit locations that nonlinearly affect the expression for δ. Assuming these variables follow *i.i.d* uniform distributions, this event is likely to occur with a very low probability 2^{-41}. The probability that it happens for more than three colliding pairs is less than 2^{-120}. So we are always likely to have enough colliding pairs to perform the attack.

6.1 Complexity of the attack

We begin with 2^{64} encryptions with all the possible IVs to find all the colliding pairs. The filtering algorithm for 2^{51} keys takes at most 2^6 computations of δ for each key guess and so for this part of the algorithm the complexity is bounded by 2^{57} calculations of δ. We need to do a brute force search over the remaining 69 keybits which would take another 2^{69} encryptions. The total complexity is the sum of the above terms and so is dominated by the 2^{69} term. The memory complexity is bounded by the memory required to find the collisions. This can be estimated to be around $2^{64} * (121 + 64) \approx 2^{71.5}$.

6.2 Extending attack to 226 rounds

The algebraic complexity of $B^i[0]$ both in terms of the number of monomials and the number of keybits involved rises very quickly after $i = 95$, as is shown graphically in Figure 2. For $i = 96$, $B^i[0]$ is a function of 101 keybits and so any attack under 2^{80} computations seems infeasible. Therefore extending the attack to more rounds seems difficult at first glance. However, we outline a method to extend the attack to 226 (=98+128) rounds using the same attack complexities. To do so, let us look at the following lemma



(A) (B)

Figure 2: Plot of (A) # Monomials, (B) # Keybits in $B^i[0]$

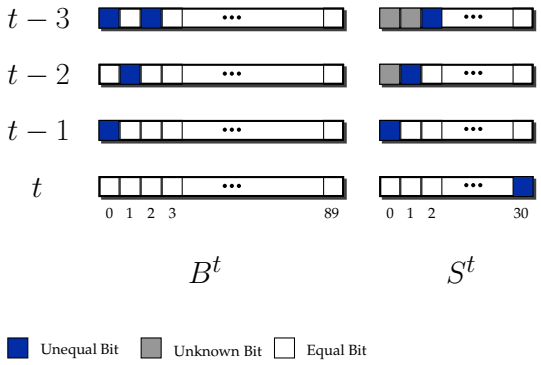


Figure 3: Backward Differential characteristic for 3 rounds in Phase 2

Lemma 1. *Let* $(B^t, S^t), (\hat{B}^t, \hat{S}^t) \in \{0,1\}^{121}$ *be two internal states in Phase 2 of* Lizard*, such that they differ only in the* 120^{th} *bit. In other words* $B^t[i] = \hat{B}^t[i]$, $\forall\ i \in [0, 89]$, $S^t[i] = \hat{S}^t[i]$, $\forall\ i \in [0, 29]$ *and* $S^t[30] = \hat{S}^t[30] \oplus 1$. *Then* $B^{t-3}[0] = \hat{B}^{t-3}[0] \oplus 1$ *with probability equal to 1. Thus the inequality of the last bit in round* t *is reflected with probability 1, in the 0th bit in round* $t-3$.

Proof. The proof is straightforward and can be obtained by running the $\mathrm{P2}^{-1}$ algorithm on (B^t, S^t) and $(\hat{B}^t, \hat{S}^t)$ three times, or by an analysis of the differential trails of the cipher. In Figure 3, we present a backward differential characteristic for 3 rounds in Phase 2. It is clear to see that the proof holds. □

The above result can be used to extend the attack to upto 98 rounds of Phase 1. We know that for an IV collision to occur for $t = 98$ rounds, the differential characteristic in the states initialized with IV_0 and IV_1 must be $0^{120}||1$. Lemma 1 implies that for this to happen, at round $t-3$ i.e. round 95, the 0^{th} bits must have a probability 1 difference, which is to say that $B^{95} \oplus \hat{B}^{95}[0] = 1$ with probability 1. In the previous attack we had utilized the fact that for Phase 2 reduced to 95 rounds, collision implies $B^{95} \oplus \hat{B}^{95}[0] = 0$ with probability 1. Thus we can repeat the previous attack with the following slight difference.

1. Compute $\delta = B^{95}[0] \oplus \hat{B}^{95}[0]$ for some 51-bit keyguess.

2. Instead of $\delta = 1$, reject the key if $\delta = 0$ and start with another key guess **else** try out another colliding IV pair.

This gives us an attack on Phase 2 reduced to 98 rounds and so 226 rounds in total. The attack complexities are exactly the same as in the previous attack.

7 Conclusion

In this paper we present a study of the stream cipher Lizard. In the first part we show that it is possible, with some effort, to find distinct key-IV pairs that produce identical keystream bits. Thereafter we construct a distinguisher for Lizard based on IVs that produce shifted keystream sequences. Finally we propose two key recovery attack on Lizard with 223 and 226 initialization rounds. The attack is similar to impossible differential attacks on block ciphers, and makes use of sparse key-IV mixing up to 95 (resp. 98) rounds of the Phase 2 initialization in the cipher.

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References

[ÅHJM11] Martin Ågren, Martin Hell, Thomas Johansson, and Willi Meier. Grain-128a: a new version of Grain-128 with optional authentication. *IJWMC*, 5(1):48–59, 2011.

[AHMNP13] Jean-Philippe Aumasson, Luca Henzen, Willi Meier, and María Naya-Plasencia. Quark: A Lightweight Hash. *J. Cryptology*, 26(2):313–339, 2013.

[AM15] Frederik Armknecht and Vasily Mikhalev. On Lightweight Stream Ciphers with Shorter Internal States. In Gregor Leander, editor, *FSE*, volume 9054 of *Lecture Notes in Computer Science*, pages 451–470. Springer, 2015.

[Ban15] Subhadeep Banik. Some Results on Sprout. In Alex Biryukov and Vipul Goyal, editors, *INDOCRYPT*, volume 9462 of *Lecture Notes in Computer Science*, pages 124–139. Springer, 2015.

[BD08] Steve Babbage and Matthew Dodd. The MICKEY Stream Ciphers. In Matthew J. B. Robshaw and Olivier Billet, editors, *The eSTREAM Finalists*, volume 4986 of *Lecture Notes in Computer Science*, pages 191–209. Springer, 2008.

[BS00] Alex Biryukov and Adi Shamir. Cryptanalytic Time/Memory/Data Tradeoffs for Stream Ciphers. In Tatsuaki Okamoto, editor, *ASIACRYPT*, volume 1976 of *Lecture Notes in Computer Science*, pages 1–13. Springer, 2000.

[CGN06] Donghoon Chang, Kishan Chand Gupta, and Mridul Nandi. RC4-Hash: A New Hash Function Based on RC4. In Rana Barua and Tanja Lange, editors, *INDOCRYPT*, volume 4329 of *Lecture Notes in Computer Science*, pages 80–94. Springer, 2006.

[CP08] Christophe De Cannière and Bart Preneel. Trivium. In Matthew J. B. Robshaw and Olivier Billet, editors, *The eSTREAM Finalists*, volume 4986 of *Lecture Notes in Computer Science*, pages 244–266. Springer, 2008.

[Dev17] The Sage Developers. *SageMath, the Sage Mathematics Software System (Version 8.0)*, 2017. `http://www.sagemath.org`.

[EK15] Muhammed F. Esgin and Orhun Kara. Practical Cryptanalysis of Full Sprout with TMD Tradeoff Attacks. In Orr Dunkelman and Liam Keliher, editors, *SAC*, volume 9566 of *Lecture Notes in Computer Science*, pages 67–85. Springer, 2015.

[est08] The ECRYPT Stream Cipher Project. *eSTREAM Portfolio of Stream Ciphers.*, September 2008.

[Fre82] Harold Fredricksen. A survey of full length nonlinear shift register cycle algorithms. *SIAM Review*, 24(2):195–221, April 1982.

[HJM07] Martin Hell, Thomas Johansson, and Willi Meier. Grain: a stream cipher for constrained environments. *IJWMC*, 2(1):86–93, 2007.

[HJMM06] Martin Hell, Thomas Johansson, Alexander Maximov, and Willi Meier. A Stream Cipher Proposal: Grain-128. In *2006 IEEE International Symposium on Information Theory*, pages 1614–1618, July 2006.

[HKM17] Matthias Hamann, Matthias Krause, and Willi Meier. LIZARD - A Lightweight Stream Cipher for Power-constrained Devices. *IACR Trans. Symmetric Cryptol.*, 2017(1):45–79, 2017.

[LNP15] Virginie Lallemand and María Naya-Plasencia. Cryptanalysis of Full Sprout. In Rosario Gennaro and Matthew Robshaw, editors, *CRYPTO (1)*, volume 9215 of *Lecture Notes in Computer Science*, pages 663–682. Springer, 2015.

[MAM16] Vasily Mikhalev, Frederik Armknecht, and Christian Müller. On Ciphers that Continuously Access the Non-Volatile Key. *IACR Trans. Symmetric Cryptol.*, 2016(2):52–79, 2016.

[RS16] Ronald L. Rivest and Jacob C. N. Schuldt. Spritz - a spongy RC4-like stream cipher and hash function. *IACR Cryptology ePrint Archive*, 2016:856, 2016.

[ZG15] Bin Zhang and Xinxin Gong. Another Tradeoff Attack on Sprout-Like Stream Ciphers. In Tetsu Iwata and Jung Hee Cheon, editors, *ASIACRYPT (2)*, volume 9453 of *Lecture Notes in Computer Science*, pages 561–585. Springer, 2015.