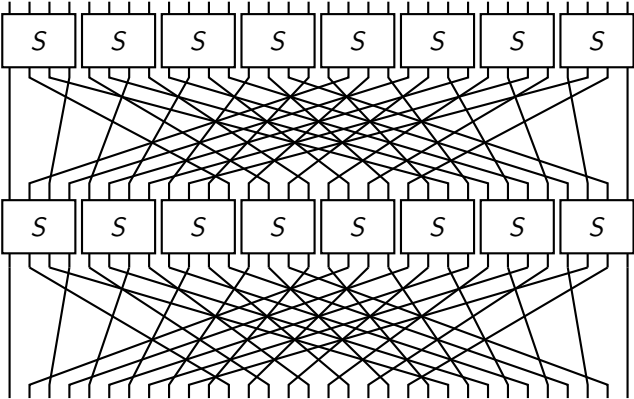


Column Parity Mixers

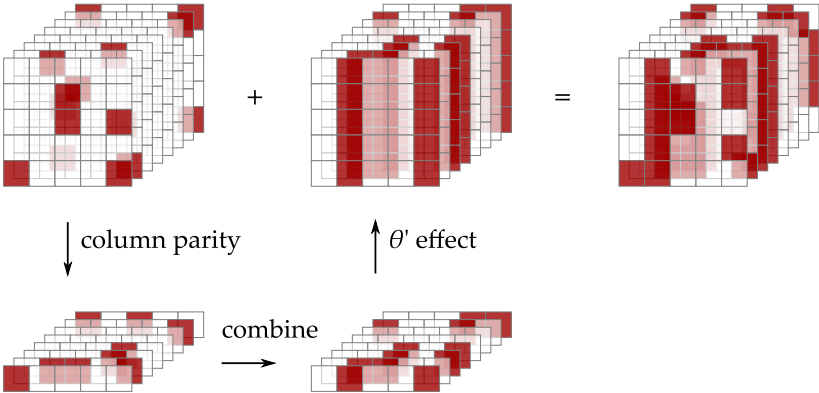
Ko Stoffelen and Joan Daemen



Diffusion



Diffusion in Keccak-f



Only 2 XORs/bit + good bounds on differential trails [MDA17]



Column parity mixers

For an $m \times n$ matrix A over \mathbb{F}_2^ℓ :

$$\theta(A) = A + f(A)$$

$$\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}$$



Column parity mixers

For an $m \times n$ matrix A over \mathbb{F}_2^ℓ :

$$\theta(A) = A + \mathbf{1}_m^T A$$

$$\underbrace{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}}_{1 \times n \text{ column parity}}$$



Column parity mixers

For an $m \times n$ matrix A over \mathbb{F}_2^ℓ :

$$\theta(A) = A + \mathbf{1}_m^T A Z$$

$$\underbrace{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}}_{1 \times n \text{ column parity}} \underbrace{\begin{pmatrix} z_{0,0} & z_{0,1} & z_{0,2} & z_{0,3} \\ z_{1,0} & z_{1,1} & z_{1,2} & z_{1,3} \\ z_{2,0} & z_{2,1} & z_{2,2} & z_{2,3} \\ z_{3,0} & z_{3,1} & z_{3,2} & z_{3,3} \end{pmatrix}}_{n \times n \text{ parity-folding matrix}} \\ \underbrace{\hspace{15em}}_{1 \times n \theta\text{-effect}}$$



Column parity mixers

For an $m \times n$ matrix A over \mathbb{F}_2^ℓ :

$$\theta(A) = A + \mathbf{1}_m \mathbf{1}_m^T A Z$$

$$\begin{array}{c} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix} \begin{pmatrix} z_{0,0} & z_{0,1} & z_{0,2} & z_{0,3} \\ z_{1,0} & z_{1,1} & z_{1,2} & z_{1,3} \\ z_{2,0} & z_{2,1} & z_{2,2} & z_{2,3} \\ z_{3,0} & z_{3,1} & z_{3,2} & z_{3,3} \end{pmatrix} \\ \underbrace{\hspace{15em}}_{1 \times n \text{ column parity}} \quad \underbrace{\hspace{15em}}_{n \times n \text{ parity-folding matrix}} \\ \underbrace{\hspace{25em}}_{1 \times n \theta\text{-effect}} \\ \underbrace{\hspace{30em}}_{m \times n \text{ expanded } \theta\text{-effect}} \end{array}$$



Column parity mixers

For an $m \times n$ matrix A over \mathbb{F}_2^ℓ :

$$\theta(A) = A + \mathbf{1}_m^m A Z$$

$$\begin{array}{c} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix} \begin{pmatrix} z_{0,0} & z_{0,1} & z_{0,2} & z_{0,3} \\ z_{1,0} & z_{1,1} & z_{1,2} & z_{1,3} \\ z_{2,0} & z_{2,1} & z_{2,2} & z_{2,3} \\ z_{3,0} & z_{3,1} & z_{3,2} & z_{3,3} \end{pmatrix} \\ \underbrace{\hspace{15em}}_{1 \times n \text{ column parity}} \quad \underbrace{\hspace{15em}}_{n \times n \text{ parity-folding matrix}} \\ \underbrace{\hspace{25em}}_{1 \times n \theta\text{-effect}} \\ \underbrace{\hspace{30em}}_{m \times n \text{ expanded } \theta\text{-effect}} \end{array}$$



Column parity mixers

For an $m \times n$ matrix A over \mathbb{F}_2^ℓ :

$$\theta(A) = A + \mathbf{1}_m^m AZ$$

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θ fully defined by m , n and Z



Special case: circulant Z

$$\begin{pmatrix} z_0 & z_1 & z_2 & z_3 \\ z_1 & z_2 & z_3 & z_0 \\ z_2 & z_3 & z_0 & z_1 \\ z_3 & z_0 & z_1 & z_2 \end{pmatrix}$$



Special case: circulant Z

$$\begin{pmatrix} z_0 & z_1 & z_2 & z_3 \\ z_1 & z_2 & z_3 & z_0 \\ z_2 & z_3 & z_0 & z_1 \\ z_3 & z_0 & z_1 & z_2 \end{pmatrix}$$

$$z(x) = z_0 + z_1x + z_2x^2 + z_3x^3$$



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$$\begin{pmatrix} z_0 & z_1 & z_2 & z_3 \\ z_1 & z_2 & z_3 & z_0 \\ z_2 & z_3 & z_0 & z_1 \\ z_3 & z_0 & z_1 & z_2 \end{pmatrix}$$

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$$\theta\text{-effect: } z(x)p(x) \bmod 1 + x^n$$



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$$z(x) = z_0 + z_1x + z_2x^2 + z_3x^3$$

$$\theta\text{-effect: } z(x)p(x) \bmod 1 + x^n$$

$$\theta(a(x, y)) = a(x, y) + \frac{1 + y^m}{1 + y} z(x)a(x, y) \bmod (1 + x^n)(1 + y^m)$$



Algebraic properties

$$\begin{aligned}\theta'(\theta(A)) &= \theta'(A + \mathbf{1}_m^m AZ) \\ &= A + \mathbf{1}_m^m AZ + \mathbf{1}_m^m AZ' + (\mathbf{1}_m^m)^2 AZZ'\end{aligned}$$



Algebraic properties

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- If m even, $(\mathbf{1}_m^m)^2 = \mathbf{0}_m^m$:
 - $\theta'(\theta(A)) = A + \mathbf{1}_m^m A(Z + Z')$
 - Group isomorphic to $(\mathbb{Z}_2^{n^2}, +)$
 - CPM is invertible, involution, commutative



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 - CPM is invertible, involution, commutative
- If m odd, $(\mathbf{1}_m^m)^2 = \mathbf{1}_m^m$:
 - $\theta'(\theta(A)) = A + \mathbf{1}_m^m A((Z + \mathbf{I})(Z' + \mathbf{I}) + \mathbf{I})$
 - Group isomorphic to $GL(n, 2)$
 - CPM is invertible iff $Z + \mathbf{I}$ is, non-commutative



Propagation properties

- Differences

$A_{\Delta} = A + A'$ at the input

$\Rightarrow B_{\Delta} = \theta(A) + \theta(A') = \theta(A_{\Delta})$ at the output



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$\Rightarrow B_{\Delta} = \theta(A) + \theta(A') = \theta(A_{\Delta})$ at the output

- Linear masks

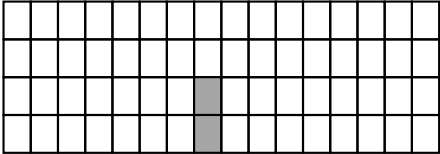
V at the output

$\Rightarrow U = V + \mathbf{1}_m^m VZ^T$ at the input



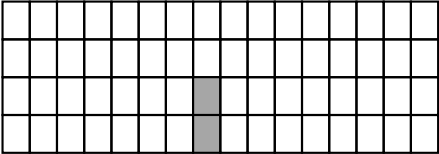
Diffusion with CPMs

- How about a state like this?



Diffusion with CPMs

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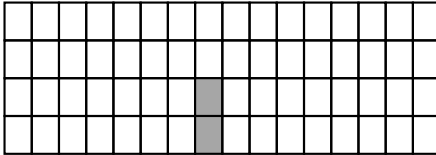


- *Orbital*: pair of active bits in the same column



Diffusion with CPMs

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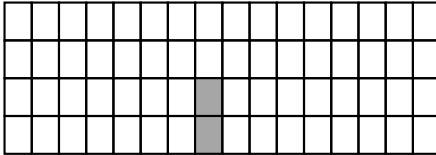


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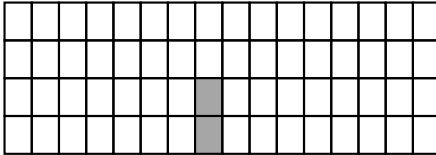


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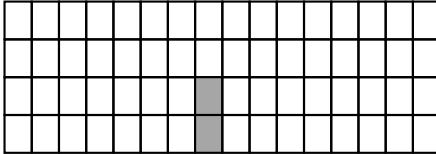


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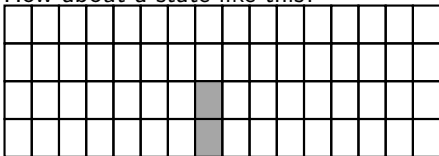


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Diffusion with CPMs

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- *Orbital*: pair of active bits in the same column
- θ is identity for states in the kernel
- States in the kernel can be expressed as a set of orbitals
- Branch number 4
- Requires transposition layer
- Single-bit difference propagates to $1 + |Z| m$ bits



CPMs vs. (near-)MDS matrices

Cipher	Type	XORs/bit	Branch no.
AES	MDS	3.03	5
Joltik	MDS	3	5
PHOTON	MDS	5 [†]	7
Prøst	MDS	4.5 [†]	5
Midori	Not MDS [‡]	1.5	4
Minalpher	Not MDS [‡]	1.5	4
Prince	Not MDS	1.5	4
SKINNY	Not MDS	0.75	2
Keccak- <i>f</i>	CPM	2	4
Circulant CPM	CPM	$2 + \frac{ z(x) -2}{m}$ *	4

* $\text{XORs/bit} \in [2 - 1/m, 2 + (n - 2)/m]$

† Unknown whether it can be computed with less XORs

‡ Can also be considered to be a CPM!



CPM example

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$



CPM example

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \\ \Leftrightarrow \\ m = 2, Z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



Building a permutation with a CPM

1. Determine design goals



Building a permutation with a CPM

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2. Pick m , n , and cell width



Building a permutation with a CPM

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7. Determine 'good' Z



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8. Pick 'good' round constants to beat all kinds of invariant attacks [BCLR17]
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10. Determine the number of rounds
11. Implement it
12. Give it a name



(Truncated) trail search

- r -round trail with weight W has differential with weight $L \leq \lfloor \frac{W}{r} \rfloor$



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- CPM causes heavy search space branching
- Dedicated software for CPM-based ciphers/permutations



Mixifer

- 16 rounds $(\iota \circ \rho \circ \pi \circ \theta \circ \gamma)$, $4 \times 16 \times 4 = 256$ bits permutation



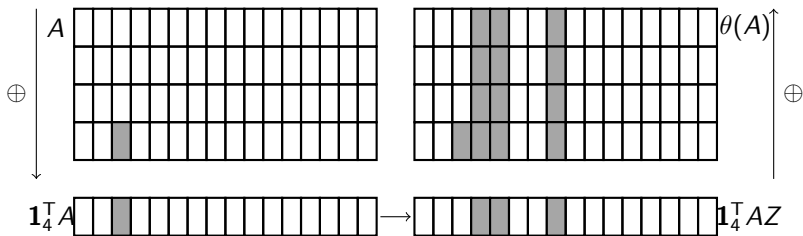
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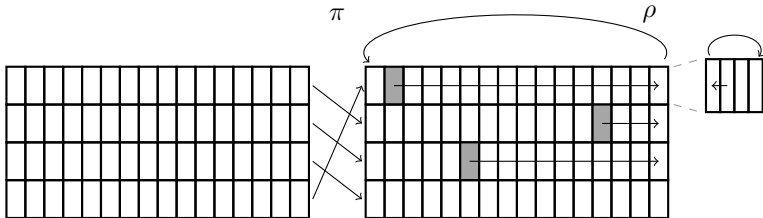
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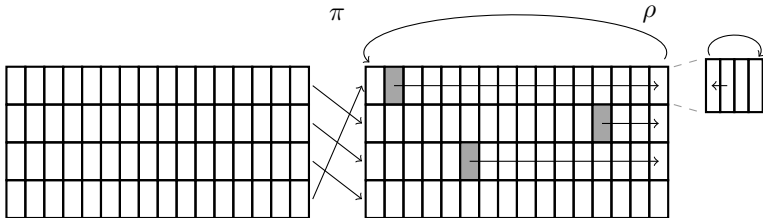
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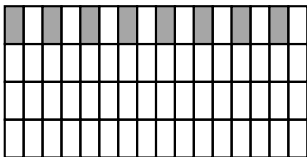
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- ι : add $0xF3485763 \ggg i$ in round i to every other cell of top row



Mixifer analysis

- Strict avalanche criterion after 3 rounds, full diffusion after 5



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- After 4 rounds:



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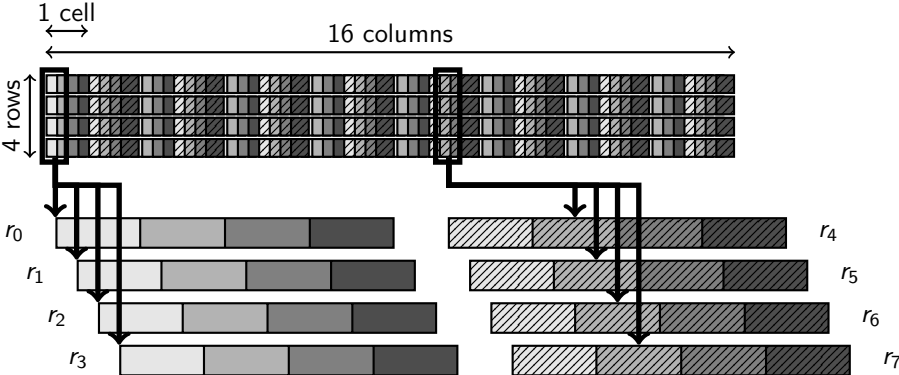


Mixifer analysis

- Strict avalanche criterion after 3 rounds, full diffusion after 5
- After 4 rounds:
 - In kernel: ≥ 52 active cells
 - Outside kernel: ≥ 46 active cells (differential), DP 2^{-92}
 - Outside kernel: ≥ 40 active cells (linear), LP 2^{-80}
- Preliminary study makes us believe trail clustering, impossible differentials, invariant attacks are not a concern



Mixer implementation



Mixer comparison (ARM Cortex-M4)

Cipher	Width (bits)	r	Speed (cpb)		Bound trails		
			Full	/ r	r	W	/ r
AES bitsliced	128	10	50.52	5.05	4	150	37.5
AES tables			39.97	4.00			
Gimli	384	24	21.81	0.91	8	52	6.5
Keccak- f [400]	400	20	106	5.3	6	92	15.3
Keccak- f [800]	800	22	48.02	2.18	6	92	15.3
Salsa20/20	512	20	13.88	0.69	3	18	6
Mixer	256	16	36.69	2.33	4	92	23



Thanks...

...for your attention

Questions?



References I



Christof Beierle, Anne Canteaut, Gregor Leander, and Yann Rotella.

Proving resistance against invariant attacks: How to choose the round constants.

In Jonathan Katz and Hovav Shacham, editors, *Advances in Cryptology – CRYPTO 2017, Part II*, volume 10402 of *Lecture Notes in Computer Science*, pages 647–678, Santa Barbara, CA, USA, August 20–24, 2017. Springer, Heidelberg, Germany.



Silvia Mella, Joan Daemen, and Gilles Van Assche.

New techniques for trail bounds and application to differential trails in Keccak.

IACR Transactions on Symmetric Cryptology, 2017(1):329–357, 2017.

